Period 1, Feb 28, 2025

17) $\int \frac{x+7}{x^2+8x+25}\,dx = \int \frac{x+4}{x^2+8x+25}\,dx + \int \frac{3}{x^2+8x+25}\,dx$

$U = x^2+8x+25$

$dU = 2x+8 = 2(x+4)$

$U = x^2+8x+25$

$dU = (2x+8)dx$

$\frac{dU}{2(x+4)} = dx$

$\int \frac{\cancel{x+4}}{U} \cdot \frac{dU}{2(\cancel{x+4})}$

$\frac{1}{2}\int \frac{dU}{U}$

$\frac{1}{2}\ln|x^2+8x+25|$

$+$

$a=1$

$b=8$

$\frac{b}{2}=4$

$\left(\frac{b}{2}\right)^2 = 4^2 = 16$

$\int \frac{3}{\underbrace{x^2+8x+16}\ \underbrace{-16+25}}\,dx$

$\int \frac{3}{(x+4)^2+9}\,dx$

$\int \frac{3}{(x+4)^2+3^2}\,dx$

$U = x+4$

$dU = dx$

$3\int \frac{1}{U^2+3^2}\,dx$

$3 \cdot \frac{1}{3}\arctan\frac{U}{3} + C$

2. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

12) $\int x^2(\sqrt{1+5x})\,dx$

$U = 1+5x \Rightarrow \frac{U-1}{5} = x$

$dU = 5dx$

$\frac{dU}{5} = dx$

$\int \left(\frac{U-1}{5}\right)^2 \cdot U^{\frac{1}{2}} \cdot \frac{dU}{5}$

$\int \frac{(U^2-2U+1)U^{\frac{1}{2}}}{25} \cdot \frac{dU}{5} = \frac{1}{125}\int (U^2+2U+1)U^{\frac{1}{2}}\,dU$

$\frac{1}{25}\int \left(U^{\frac{5}{2}} + 2U^{\frac{3}{2}} + U^{\frac{1}{2}}\right)dU$

16) $\int \frac{7}{\sqrt{25-81x^2}}dx = \int \frac{7}{\sqrt{5^2-(9x)^2}}dx = 7\int \frac{dx}{\sqrt{5^2-(9x)^2}}$

1. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$

$u = 9x$

$du = 9dx$

$\frac{du}{9} = dx$

$7\int \frac{1}{9}\cdot\frac{du}{\sqrt{5^2-u^2}}$

$\frac{7}{9}\int \frac{du}{\sqrt{5^2-u^2}}$

13) $\int_1^{13} \frac{x}{\sqrt{2x-1}}dx$

$u = 2x-1 \Rightarrow \frac{u+1}{2} = x$

$du = 2dx$

$\frac{du}{2} = dx$

$2(1)-1=1$

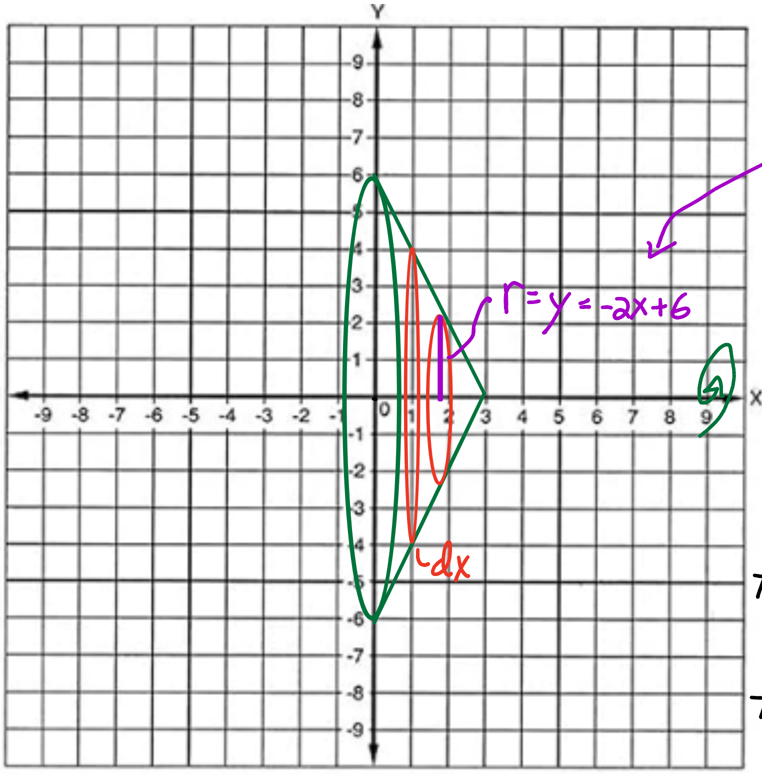
$2(13)-1=25$

$\int_1^{25} \frac{\frac{u+1}{2}}{\frac{\sqrt{u}}{1}}\cdot\frac{du}{2}$

$\int_1^{25} \frac{u+1}{2}\cdot\frac{1}{\sqrt{u}}\cdot\frac{du}{2} = \frac{1}{4}\int_1^{25}\frac{u+1}{u^{\frac{1}{2}}}du$

$\frac{1}{4}\int_1^{25}\left[\frac{u}{u^{\frac{1}{2}}}+\frac{1}{u^{\frac{1}{2}}}\right]du$

$\frac{1}{4}\int_1^{25}\left[u^{\frac{1}{2}}+u^{-\frac{1}{2}}\right]du$



$y = -2x + 6$

in Quad I

Rotate around x-axis

Volume $\underline{dx}$

$\pi r^2 h$

$$\int_0^3 \pi(-2x+6)^2 \cdot dx$$

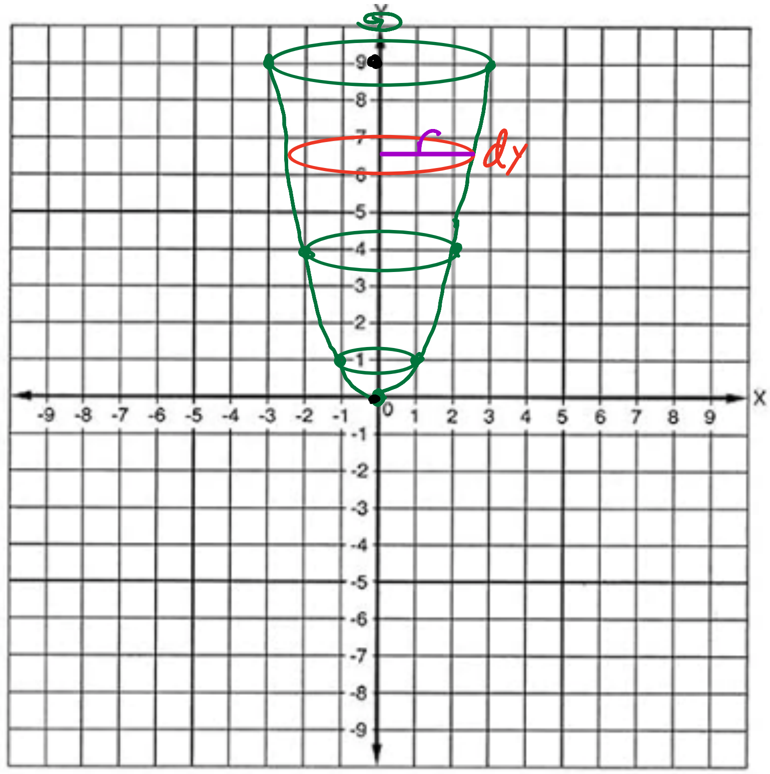
$$\pi \int_0^3 (4x^2 - 24x + 36)\, dx$$

$$\pi\left(\frac{4}{3}x^3 - 12x^2 + 36x\right)\Big[_0^3$$

$36\pi \text{ in}^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$\cancel{\tfrac{1}{3}} \cdot \pi \cdot 6^2 \cdot \cancel{3} = 36\pi$$



$y = x^2 \Rightarrow x = \sqrt{y}$

$0 \le x \le 3$

Rotate around y-axis

Volume?

$\pi r^2 h$ = disc method

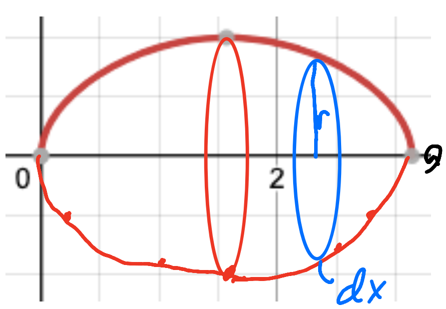
$\pi r^2 \, dy$

$r = x$

$$\int_0^9 \pi (x)^2 \cdot dy$$

$$\int_0^9 \pi y \, dy = \frac{\pi}{2} y^2 + C \Big|_0^9$$

$\frac{81\pi}{2}$ units^3



$y = \sqrt{\sin x}$

$r = y = \sqrt{\sin x}$

Volume of disc $= \pi r^2 h = \pi (\sqrt{\sin x})^2 \cdot dx$

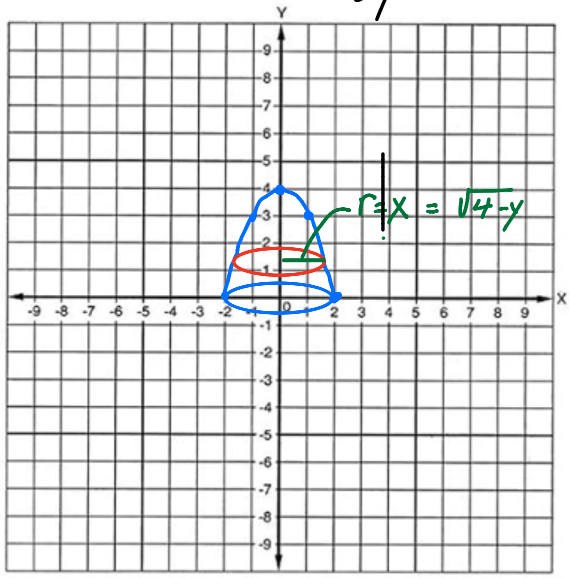
$$\int_0^\pi \pi \sin x \, dx = -\pi \cos x \Big|_0^\pi$$

$-\pi \cos \pi - (-\pi \cos 0) =$ Volume

$-\pi(-1) - (-\pi(1)) =$

$\pi + \pi = 2\pi$

$\pi r^2 \cdot dy$



$y = 4 - x^2$ in Quad I

x	y
0	4
1	3
2	0

Rotate around

a) the y-axis

Volume $\displaystyle\int_0^4 \pi(\sqrt{4-y})^2\,dy$

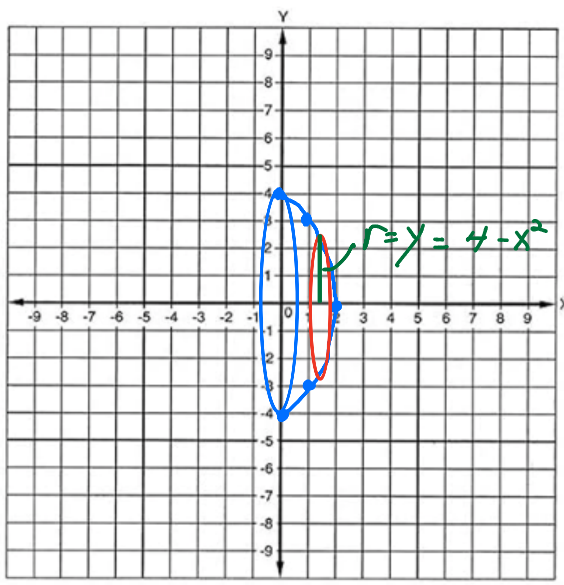
$\displaystyle\pi\int_0^4 (4-y)\,dy$

$\pi\left[4y - \frac{1}{2}y^2\right]\Big|_0^4$

$\pi\left[4\cdot4 - \frac{1}{2}(4)^2\right] - \pi\left[4\cdot 0 - \frac{1}{2}(0)^2\right]$

$\pi[16-8] = 8\pi$

Rotate around x-axis



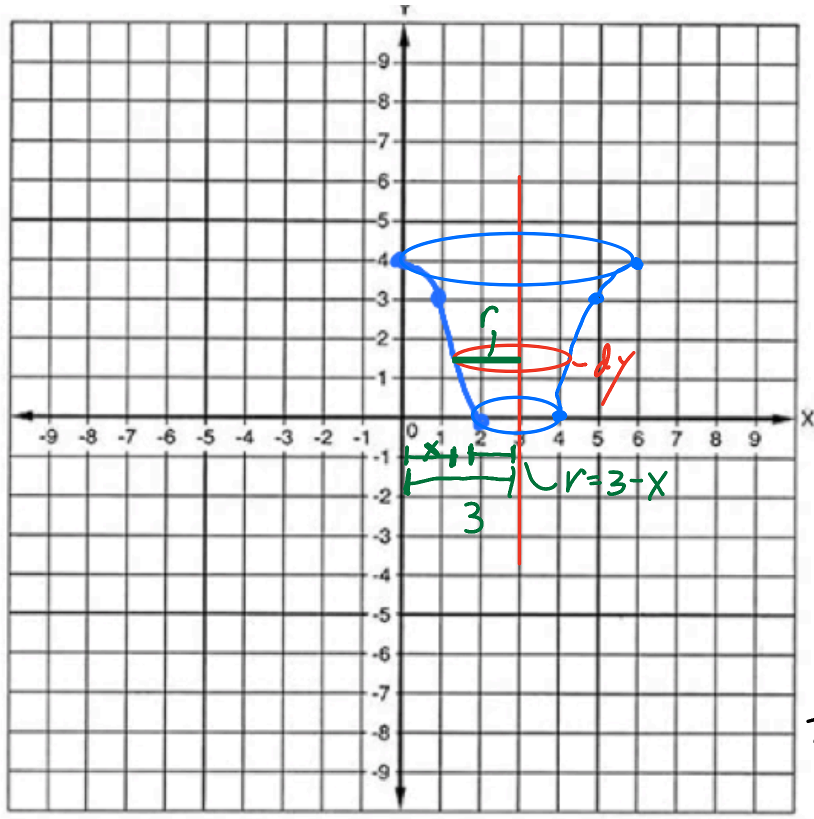
$\displaystyle\int_0^2 \pi(4-x^2)^2\,dx$

$\displaystyle\pi\int_0^2 (16 - 8x^2 + x^4)\,dx$

$\pi\left(16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right)\Big|_0^2$

$\pi\left(16(2) - \frac{8}{3}(2)^3 + \frac{1}{5}(2)^5\right) - \pi\left(16(0) - \frac{8}{3}(0)^3 + \frac{1}{5}(0)^5\right)$

$\pi\left(32 - \frac{64}{3} + \frac{32}{5}\right)$



Rotate around $x=3$ Find Volume

$$y = 4 - x^2 \Rightarrow x = \sqrt{4-y}$$

$$\int_0^4 \pi (3-x)^2 \, dy$$

$$\int_0^4 \pi \left(3 - \sqrt{4-y}\right)^2 dy$$

$$\pi \int_0^4 \left(9 - 6\sqrt{4-y} + 4 - y\right) dy$$

$$\pi \left[9y + 4y - \frac{1}{2}y^2\right]\Big|_0^{4} - \int_0^4 6\sqrt{4-y}\, dy$$